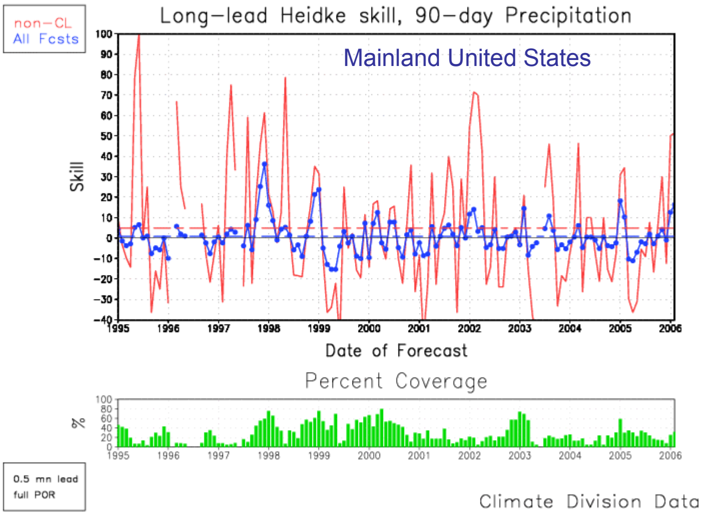
Verification of climate predictions

The Heidke skill score (a “hit score”)



Heidke Skill Score (for deterministic categorical forecasts)

$$\text{Heidke score} = 100\left(\frac{\#Hits - \#Expected}{\#Total - \#Expected}\right)$$

Example: Suppose for OND 1997, rainfall forecasts are made for 15 stations in southern Brazil. Suppose forecast is defined by tercile-based category having highest probability. Suppose for all 15 stations, “above” is forecast with highest probability, and that observations were above normal for 12 stations, and near normal for 3 stations. Then Heidke score is:

100 X (12 – 15/3) / (15 – 15/3)

100 X 7 / 10

= 70

Note that the probabilities given in the forecasts did not matter, only which category had highest probability.

Credit/Penalty matrix for some Variations of the Heidke Skill Score

FORECAST

OBSERVATION

	Below	Near	Above
Below	1 (0.67)	0 (-0.33)	0 (-0.33)
Near	0 (-0.33)	1 (0.67)	0 (-0.33)
Above	0 (-0.33)	0 (-0.33)	1 (0.67)

Original Heidke score (Heidke, 1926 [in German])

OBSERVATION

	Below	Near	Above
Below	1.125	0	-1.125
Near	-0.375	0.750	-0.375
Above	-1.125	0	1.125

Modified in Barnston (Wea. and Forecasting, 1992)

OBSERVATION

	Below	Near	Above
Below	0.89	-0.11	-0.78
Near	-0.11	0.22	-0.11
Above	-0.78	-0.11	0.89

LEPS for Terciles (Potts et al., J. Climate,1996

Root-mean-Square Skill Score: RMSSS for continuous deterministic forecasts

RMSSS is defined as: $RMSSS = 100 \times \left(1 - \left(\frac{RMS_f}{RMS_s}\right)\right)$

where: $RMSE_f$ = root mean square error of forecasts, and $RMSE_s$ = root mean square error of standard used as no-skill baseline.

Both persistence and climatology can be used as baseline. Persistence, for a given parameter, is the persisted anomaly from the forecast period immediately prior to the LRF period being verified. For example, for seasonal forecasts, persistence is the seasonal anomaly from the season period prior to the season being verified. Climatology is equivalent to persisting an anomaly of zero.

RMS_f = $\sqrt{\frac{\sum_{i=1}^{N}\left[\left(f_i - O_i\right)^2 W_i\right]}{\sum_{i=1}^{N} W_i}}$

$$\mathrm{RMS}_f = \sqrt{\frac{\sum_{i=1}^{N}\left[\left(f_i - O_i\right)^2 W_i\right]}{\sum_{i=1}^{N} W_i}}$$

where: i stands for a particular location (grid point or station).

f_i = forecasted anomaly at location i
O_i = observed or analyzed anomaly at location i.

W_i = weight at grid point i, when verification is done on a grid, set by W_i = cos(latitude)

N = total number of grid points or stations where verification is carried.

RMSSS is given as a percentage, while RMS scores for f and for s are given in the same units as the verified parameter.

The RMS and the RMSSS are made larger by three main factors:

(1) The mean bias
(2) The conditional bias
(3) The correlation between forecast and obs

It is easy to correct for (1) using a hindcast history. This will improve the score. In some cases (2) can also be removed, or at least decreased, and this will improve the RMS and the RMSSS farther. Improving (1) and (2) does not improve (3). It is most difficult to increase (3). If the tool is a dynamical model, a spatial MOS correction can increase (3), and help improve RMS and RMSSS.

Murphy (1988), Mon. Wea. Rev.

Verification of Probabilistic Categorical Forecasts: The **Ranked Probability Skill Score (RPSS)**

Epstein (1969), J. Appl. Meteor.

RPSS measures cumulative squared error between categorical forecast probabilities and the observed categorical probabilities relative to a reference (or standard baseline) forecast.

The observed categorical probabilities are 100% in the observed category, and 0% in all other categories.

$$RPS = \sum_{cat=1}^{Ncat} (Pcum_{F(cat)} - Pcum_{O(cat)})^2$$

Where Ncat = 3 for tercile forecasts. The “cum” implies that the summation is done for cat 1, then cat 1 and 2, then cat 1 and 2 and 3.

$$RPS = \sum_{cat=1}^{Ncat} (Pcum_{F(cat)} - Pcum_{O(cat)})^2$$

The higher the RPS, the poorer the forecast. RPS=0 means that the probability was 100% given to the category that was observed.

The RPSS is the RPS for the forecast compared to the RPS for a reference forecast that gave, for example, climatological probabilities.

$$RPSS = 1 - \frac{RPS_{forecast}}{RPS_{reference}}$$

RPSS > 0 when RPS for actual forecast is smaller than RPS for the reference forecast.

Suppose that the probabilities for the 15 stations in OND 1997 in Southern Brazil, and the observations were:

	forecast(%)	obs(%)	RPS calculation
1	20 30 50	0 0 100	RPS=$(0-.20)^2+(0-.50)^2+(1.-1.)^2$ =.04+.25 +.0 = .29
2	25 35 40	0 0 100	RPS=$(0-.25)^2+(0-.60)^2+(1.-1.)^2$ =.06+.36 +.0 = .42
3	25 35 40	0 0 100	
4	20 35 45	0 0 100	RPS=$(0-.20)^2+(0-.55)^2+(1.-1.)^2$ =.04+.30 +.0 = .34
5	15 30 55	0 0 100	
6	25 35 40	0 0 100	
7	25 35 40	0 100 0	RPS=$(0-.25)^2+(1-.60)^2+(1.-1.)^2$ =.06+.16 +.0 = .22
8	25 35 40	0 0 100	
9	20 35 45	0 0 100	
10	25 35 40	0 0 100	
11	25 35 40	0 100 0	
12	20 35 40	0 100 0	
13	15 30 55	0 0 100	RPS=$(0-.15)^2+(0-.45)^2+(1.-1.)^2$ =.02+.20 +.0 = .22
14	25 35 40	0 0 100	
15	25 35 40	0 0 100	

Finding RPS for reference (climatol baseline) forecasts:

for 1st forecast, RPS(clim) = $(0-.33)^2+(0-.67)^2+(1.-1.)^2$ = .111+.444+0=.556

for 7th forecast, RPS(clim) = $(0-.33)^2+(1.-.67)^2+(1.-1.)^2$ = .111+.111+0=.222

for a forecast whose observation is “below” or “above”, PRS(clim)=.556

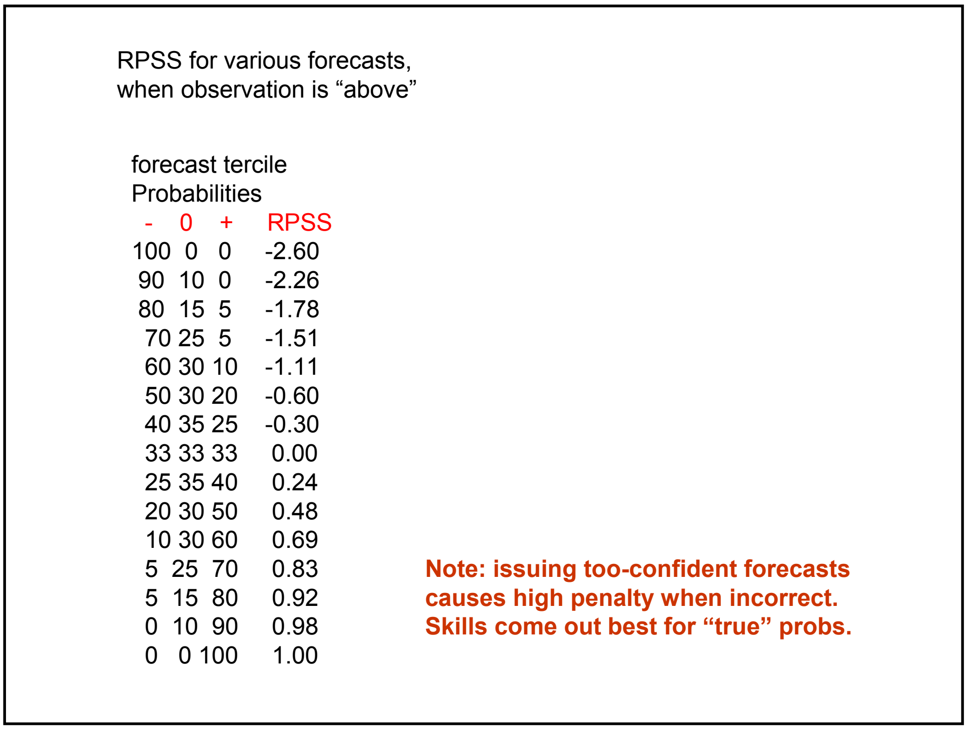
	forecast(%)	obs(%)	RPS and RPSS(clim)		RPSS
1	20 30 50	0 0 100	RPS= .29	RPS(clim)= .556	1-(.29/.556) = .48
2	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
3	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
4	20 35 45	0 0 100	RPS= .34	RPS(clim)= .556	1-(.34/.556) = .39
5	15 30 55	0 0 100	RPS= .22	RPS(clim)= .556	1-(.22/.556) = .60
6	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
7	25 35 40	0 100 0	RPS= .22	RPS(clim)= .222	1-(.22/.222) = .01
8	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
9	20 35 45	0 0 100	RPS= .34	RPS(clim)= .556	1-(.34/.556) = .39
10	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
11	25 35 40	0 100 0	RPS= .22	RPS(clim)= .222	1-(.22/.222) = .01
12	20 35 40	0 100 0	RPS= .22	RPS(clim)= .222	1-(.22/.222) = .01
13	15 30 55	0 0 100	RPS= .22	RPS(clim)= .556	1-(.22/.556) = .60
14	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24
15	25 35 40	0 0 100	RPS= .42	RPS(clim)= .556	1-(.42/.556) = .24

Finding RPS for reference (climatol baseline) forecasts:

When obs=“below”, RPS(clim) = $(0-.33)^2+(0-.67)^2+(1.-1.)^2$ =.111+.444+0=.556

When obs=“normal”, RPS(clim)=$(0-.33)^2+(1.-.67)^2+(1.-1.)^2$ =.111+.111+0=.222

When obs=“above”, RPS(clim)= $(0-.33)^2+(0-.67)^2+(1.-1.)^2$ =.111+.444+0=.556

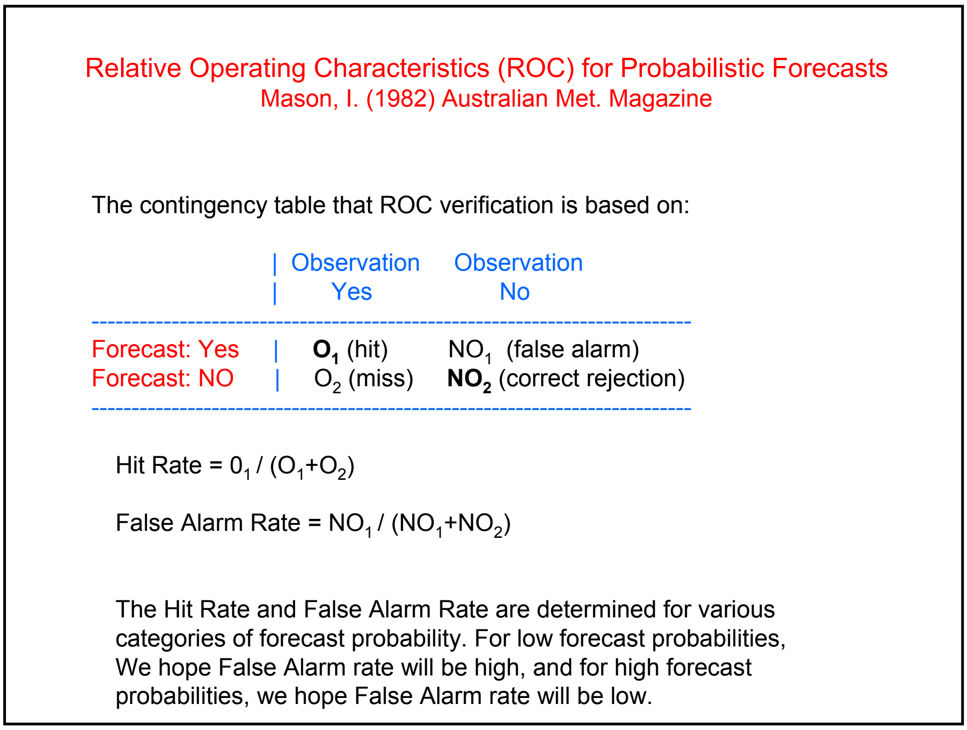


RPSS for various forecasts,
when observation is “above”

forecast tercile
Probabilities

-	0	+	RPSS
100	0	0	-2.60
90	10	0	-2.26
80	15	5	-1.78
70	25	5	-1.51
60	30	10	-1.11
50	30	20	-0.60
40	35	25	-0.30
33	33	33	0.00
25	35	40	0.24
20	30	50	0.48
10	30	60	0.69
5	25	70	0.83
5	15	80	0.92
0	10	90	0.98
0	0	100	1.00

Note: issuing too-confident forecasts causes high penalty when incorrect. Skills come out best for “true” probs.



Relative Operating Characteristics (ROC) for Probabilistic Forecasts

Mason, I. (1982) Australian Met. Magazine

The contingency table that ROC verification is based on:

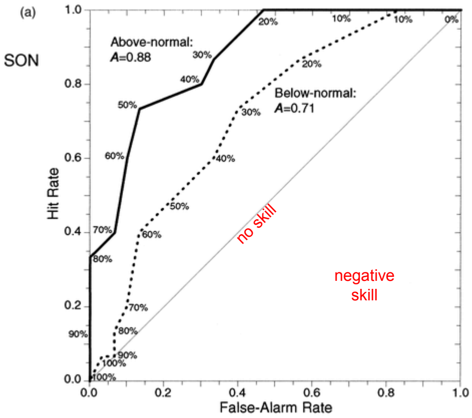
	Observation Yes	Observation No
Forecast: Yes	**O_1** (hit)	NO_1 (false alarm)
Forecast: NO	O_2 (miss)	**NO_2** (correct rejection)

Hit Rate = $0_1 / (O_1+O_2)$

False Alarm Rate = $NO_1 / (NO_1+NO_2)$

The Hit Rate and False Alarm Rate are determined for various categories of forecast probability. For low forecast probabilities, We hope False Alarm rate will be high, and for high forecast probabilities, we hope False Alarm rate will be low.

	Observation Yes	Observation No
Forecast: Yes	**O_1** (hit)	NO_1 (false alarm)
Forecast: NO	O_2 (miss)	**NO_2** (correct rejection)



The curves are cumulative from left to right. For example, "20%" really means "100% + 90% +80% + ….. +20%".

Curves farther to the upper left show greater skill.

Example from Mason and Graham (2002), QJRMS, for eastern Africa OND simulations (observed SST forcing) using ECHAM3 AGCM

	Observation Yes	Observation No
Forecast: Yes	**O_1** (hit)	NO_1 (false alarm)
Forecast: NO	O_2 (miss)	**NO_2** (correct rejection)

Hanssen and Kuipers (1965), Koninklijk Nederlands Meteorologist Institua Meded. Verhand, 81-2-15

The Hanssen and Kuipers score is derivable from the above contingency table. Hanssen and Kuipers (1965), Koninklijk Nederlands Meteorologist Institua Meded. Verhand, 81-2-15 It is defined as KS = Hit Rate - False Alarm Rate (ranges from -1 to +1, but can be scaled for 0 to +1).

$$\text{KS} = \frac{O_1 NO_2 - O_2 NO_1}{(O_1 + O_2) - (NO_1 + NO_2)}$$

When scale the KS as $\text{KS}_{\text{scaled}}$ = (KS+1) / 2 then the score is comparable to the area under the ROC curve.

n_{ij}	Observed **Below** Normal	Observed Near Normal	Observed Above Normal	
Forecast **Below** Normal	n11	n12	n13	n1■
Forecast Near Normal	n21	n22	n23	n2■
Forecast Above Normal	n31	n32	n33	n3■
	n■1	n■2	n■3	N■■ (total)

Basic input to the Gerrity Skill Score: sample contingency table.

$$probability_{ij} = p_{ij} = \frac{n_{ij}}{N}$$

Gerrity Skill Score = GSS $= \sum_{i=1}^{3} \sum_{j=1}^{3} p_{ij} s_{ij}$

Gerrity (1992), Mon. Wea. Rev.

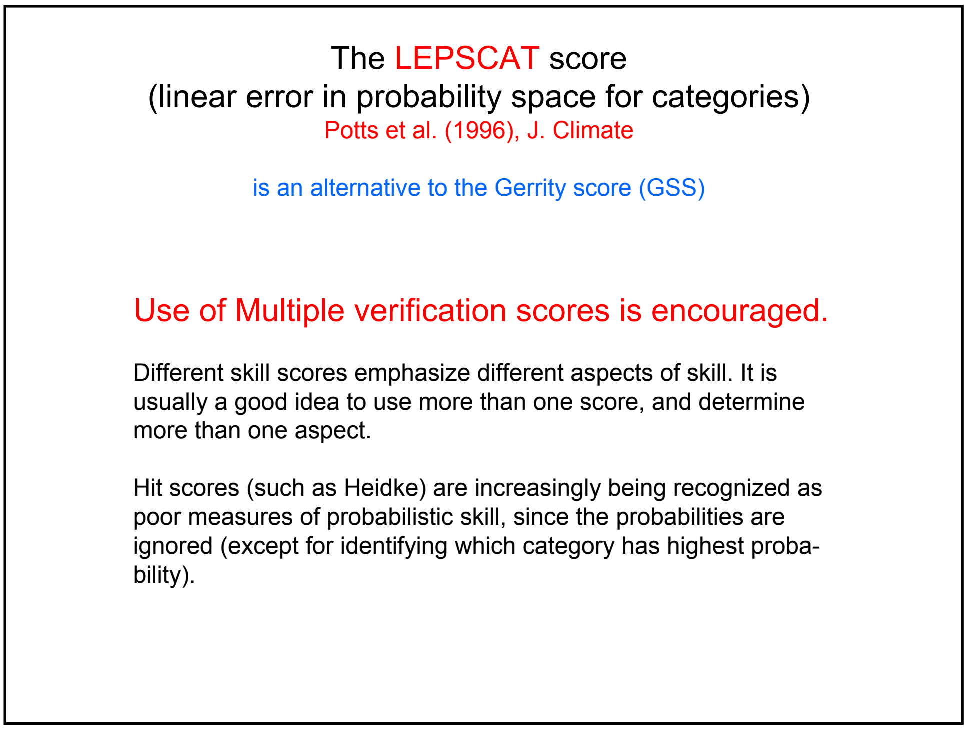
S_{ij} is the scoring matrix

$$s_{ii} = \frac{1}{2}\left(\sum_{r=1}^{i-1} a_r^{-1} + \sum_{r=i}^{2} a_r \right)$$

$$s_{ij} = \frac{1}{2}\left(\sum_{r=1}^{i-1} a_r^{-1} - (j-1) + \sum_{r=j}^{2} a_r \right) ; 1 \le i < 3, i < j \le 3$$

where $$a_i = \frac{1 - \sum_{r=1}^{i} p_r}{\sum_{r=1}^{i} p_r}$$

Note that GSS is computed using the sample probabilities, not those on which the original categorizations were based (0.333,0.333,0.333).



The LEPSCAT score (linear error in probability space for categories)

Potts et al. (1996), J. Climate

is an alternative to the Gerrity score (GSS)

Use of Multiple verification scores is encouraged.

Different skill scores emphasize different aspects of skill. It is usually a good idea to use more than one score, and determine more than one aspect.

Hit scores (such as Heidke) are increasingly being recognized as poor measures of probabilistic skill, since the probabilities are ignored (except for identifying which category has highest probability).